Given Quantities in the Light of Measurements by Applying Modern Technologies

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Abstract. By using the modern measuring technologies it is possible to obtain measurements of higher accuracy than the accuracy of given quantities to which these measurements are related. In this way the problem of error influence for given quantities on the adjusted ones arises. The paper presents how the influences of given quantities errors on the adjusted quantities are taken into account. The theory is followed by an example of adjustment and accuracy estimate for a traverse defined by given trigonometric points where the measurements are performed by use of a total station. In this example both adjustment techniques are applied, the classical and with given quantities.

Keywords: Measurement, modern technologies, successive adjustments, error, given quantities.
1 Introduction

The use of modern geodetic measuring technologies offers the possibility to obtain results of higher accuracy compared to that characterising the earlier (classical) measurements. In the case of geodetic networks these measurements must be related to the existing networks. In such a way in the adjustment and accuracy estimation, a problem arises concerning the given quantities; their errors have a larger influence on the observed-function error than the errors of the new measurements. These problems have been known since long ago. This theory was developed and applied by Russian geodesists as early as 150 years ago. In the former Yugoslavia the first who applied and developed this theory was Krunislav Mihailović in 1965 in his PhD thesis, [1]. In 2005, Perović, [2] and [3] – Chapter 19 Successive Adjustment, gave a detailed review of this theory followed by some improvements and generalisations.

This theory has been very rarely applied by geodesists, only for the networks in engineering. In the national networks it has not been applied because the national networks have been designed and formed as successive ones with negligibly small errors of the given quantities (networks). Today we have a reverse case that newly formed networks are more accurate than the given ones and, consequently, the problem of the errors of the given quantities becomes a reality. In other words, in the process of adjustment and accuracy estimation for the new networks, the errors of the given quantities must be taken into account.

2 Methodology

A consensus of specialists in geodesy all over the world exists that successive networks are to be formed under the following basic principles – conditions:

1. **Condition**: Any newly introduced network must form a unified system together with the existing network to which it is related – a unique network, and

2. **Condition**: In the adjustment of the new network the points of the given network preserve their coordinates.

A detailed description of this theory was given by Perović, [2] and [3] – Chapter 19 Successive Adjustment. If the coordinate vector for the new-network points is designated as \( \mathbf{x} \), and the coordinate vector for the given-network points as \( \mathbf{\xi} \), then the correction equations can be written as

\[
\mathbf{v} = \mathbf{A} \mathbf{x} + \mathbf{B} \mathbf{\xi} + \mathbf{f},
\]

so that the influences of the given quantities \( \mathbf{\xi} \) become conspicuous, where \( \mathbf{A} \) is the matrix of derivative values for the measurement-vector components in the coordinates \( \mathbf{x} \) with full rank of the columns, and \( \mathbf{B} (\mathbf{B} \neq \mathbf{0}) \) is the matrix of derivative values for the measurement-vector components in the coordinates \( \mathbf{\xi} \).

Equations (1) are represented as

\[
\mathbf{v} = \mathbf{A} \mathbf{x} + \mathbf{\bar{f}},
\]

with

\[
\mathbf{\bar{f}} = \mathbf{f} + \mathbf{B} \mathbf{\xi} = \mathbf{1} (\mathbf{\xi}_0, \mathbf{X}_0) + \mathbf{B} \mathbf{\xi} - \mathbf{1}.
\]

Now the influence of the errors of the given quantities \( \mathbf{\xi} \) is also included in the free-term vector \( \mathbf{\bar{f}} \) so that the stochastic model is

\[
\mathbf{M}[\mathbf{v}] = \mathbf{0}, \quad \text{with} \quad \mathbf{K}_\mathbf{v} = \mathbf{K}_\mathbf{T} = \mathbf{K}_\mathbf{T} + \mathbf{B} \mathbf{\xi} \mathbf{B}^\mathsf{T},
\]

where the matrix \( \mathbf{K}_\mathbf{T} \) is regular, [2] and [3].

The application of the least squares to (2), with (3) and (4), yields

\[
\mathbf{x} = -\mathbf{D} \mathbf{\bar{f}}, \quad \mathbf{\hat{v}} = \mathbf{A} \mathbf{x} + \mathbf{\bar{f}},
\]

\[
m_0 \mathbf{v}^2 = \frac{1}{f} \mathbf{v}^\mathsf{T} \mathbf{K}_\mathbf{T}^{-1} \mathbf{v}, \quad f = n - u,
\]

\[
\mathbf{K}_\mathbf{x} = \mathbf{N}^{-1}, \quad \mathbf{N}^{-1} = (\mathbf{A}^\mathsf{T} \mathbf{K}_\mathbf{T}^{-1} \mathbf{A}).
\]

with

\[
\mathbf{D} = (\mathbf{A}^\mathsf{T} \mathbf{K}_\mathbf{T}^{-1} \mathbf{A})^{-1} \mathbf{A}^\mathsf{T} \mathbf{K}_\mathbf{T}^{-1}.
\]

The variance-covariance matrix for the correction estimates and the observational-reliability one are given, respectively:

\[
\mathbf{K}_\mathbf{\hat{v}} = \mathbf{K}_\mathbf{T} - \mathbf{A} \mathbf{N}^{-1} \mathbf{A}^\mathsf{T},
\]

\[
\mathbf{R} = \mathbf{K}_\mathbf{\xi} \mathbf{K}_\mathbf{T}^{-1}.
\]

Provided that all other error sources are eliminated, [2] and [3] – Chapter 6.9 – Gross-Error Test:

1. **In the functional model**: (1) relationships (omitted regressors), (2) the coordinate system, (3) instruments and devices (calibration parameters), (4) gravitational field, (5) refraction model, (6) time factor. (These effects not taken into account in the model can be regarded as reductions or corrections to the observations, or they are described (absorbed) through introducing additional parameters (Perović [2] and [3] – Chapter 30 – Covariance Analysis);
2. **In the stochastic model**: (1) variance a priori, (2) correlation. (A substantially more realistic a priori variance-covariance matrix is required);

3. **In the observation data**: (1) gross error of the measurements, (2) errors in data registration, (3) blunders in error identification, (4) instability of observation pillars, (5) (gross) errors of centering. (A survey of observations and residuals combined with statistical tests can be useful in data cleaning);

4. **In the calculations**: (1) computer-program errors, (2) input errors, (3) numerical instability of a matrix inversion, (4) error accumulation due to rounding, (5) voltage reduction in computer. (Use of independent programs can indicate existence for some of these problems), only then one can apply the gross-error test.

For the Data Snooping method, the test quantities are:

a) For a global test:

\[ F = \frac{m^2}{\sigma_0^2}, \quad (12) \]

b) For local tests:

\[ t_i = \frac{\left( K_T^{-1} \hat{\mathbf{y}} \right)_i}{\sqrt{\left( K_T^{-1} \mathbf{K}_i K_T^{-1} \right)_{ii}}}, \quad i = 1, \ldots, n, \quad (13) \]

where \( \sigma_0^2 \) is a variance coefficient known a priori.

### 3 Results and Discussion

An illustration concerning the theory of given quantities will be presented through the traverse adjustment containing five points (where two known points are included), positioned on 4 given trigonometric points, T13, T19, T20 and T21 and measured with a seven-second total station TS7, of a measurement distance standard of 2 mm + 2 mm/km \((a = 2 \text{ mm}, \ b = 2 \text{ mm/km})\). The new-network points are traverse points P3, P4 and P5. The data are given in Table 1.

The vector of unknowns is \( \mathbf{x}^T = [y_3 \ x_3 \ y_4 \ x_4 \ y_5 \ x_5] \), that of the given coordinates \( \mathbf{x}^T = [y_{13} \ x_{13} \ y_{19} \ x_{19} \ y_{20} \ x_{20} \ y_{21} \ x_{21}] \). The variance-covariance matrix concerning the vector of the given coordinates can be considered as being diagonal with \( K_\xi = (50 \text{ mm})^2 \mathbf{E} \) – according to [5]. The measurement vector is \( \mathbf{l}^T = [\beta_1 \beta_3 \beta_4 \beta_5 \ D_{19-3} \ D_{3-4} \ D_{4-5} \ D_{5-20}] \).

If \( D_1 \) is the left-hand angle ray and \( D_2 \) the right-hand one, \( \sigma_{CI} \) and \( \sigma_{CS} \), respectively, the centring standards for the instrument and signal, \( \sigma_{STI} \) and \( \sigma_{STS} \), respectively, stability standards for the instrument and signal, \( \sigma_{\beta,0} = 2,7^{\prime\prime} \) – the direction-measuring standard with TS7 in one quick look, \( s = 2 \) – number of quick looks, then the variance for the angle measuring will be [4]:

\[
\sigma_{\beta}^2 = \frac{2\sigma_{\beta,0}^2}{s} + \frac{\rho^2}{2} (\sigma_{CI}^2 + \sigma_{STI}^2) \left( \frac{1}{D_1^2} - \frac{2\cos\beta}{D_1 D_2 + 1} \right) + \frac{\rho^2}{2} (\sigma_{CS}^2 + \sigma_{STS}^2) \left( \frac{1}{D_1^2} + \frac{1}{D_2^2} \right), \quad (14)
\]

whereas the variance for the length measuring will be [4]:

\[
\sigma_{D}^2 = \frac{1}{2} (\sigma_{CI}^2 + \sigma_{STI}^2) + \frac{1}{2} (\sigma_{CS}^2 + \sigma_{STS}^2) + (a + bD)^2, \quad (15)
\]

<table>
<thead>
<tr>
<th>Point</th>
<th>Y [m]</th>
<th>X [m]</th>
<th>Station</th>
<th>Left-Hand</th>
<th>Right-Hand</th>
<th>Angle Measuring</th>
<th>From</th>
<th>To</th>
<th>D [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>T13</td>
<td>837.95</td>
<td>335.60</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>T19</td>
<td>204.25</td>
<td>725.28</td>
<td>T19</td>
<td>T13</td>
<td>P3</td>
<td>270° 15’ 09’’</td>
<td>T19</td>
<td>P3</td>
<td>193.19</td>
</tr>
<tr>
<td>P3</td>
<td>306.170</td>
<td>889.398</td>
<td>P3</td>
<td>T19</td>
<td>P4</td>
<td>169° 30’ 36’’</td>
<td>P3</td>
<td>P4</td>
<td>159.32</td>
</tr>
<tr>
<td>P4</td>
<td>364.176</td>
<td>1037.783</td>
<td>P4</td>
<td>P3</td>
<td>P5</td>
<td>184° 30’ 36’’</td>
<td>P4</td>
<td>P5</td>
<td>164.20</td>
</tr>
<tr>
<td>P5</td>
<td>435.798</td>
<td>1185.539</td>
<td>P5</td>
<td>P4</td>
<td>T20</td>
<td>173° 37’ 08’’</td>
<td>P5</td>
<td>T20</td>
<td>184.68</td>
</tr>
<tr>
<td>T20</td>
<td>497.43</td>
<td>1359.66</td>
<td>T20</td>
<td>P5</td>
<td>T21</td>
<td>250° 30’ 30’’</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T21</td>
<td>1697.43</td>
<td>1359.66</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Data for traverse inserted between two given trigonometric points.
The matrices \( A \) and \( B \) are:

\[
\begin{bmatrix}
1 & y_3 & x_3 & y_4 & x_4 & y_5 & x_5 \\
\beta_{19} & 0.9070 & -0.5633 & 0 & 0 & 0 & 0 \\
\beta_3 & -2.1128 & 1.0346 & 1.2058 & -0.4714 & 0 & 0 \\
\beta_4 & 1.0269 & -0.4714 & -2.3362 & 1.0193 & 1.1304 & -0.5479 \\
\beta_5 & 0 & 0 & 1.1304 & -0.5479 & -2.1831 & 0.9206 \\
\beta_{20} & 0 & 0 & 0 & 0 & 1.0527 & -0.3726 \\
D_{19,3} & 0.5276 & 0.8495 & 0 & 0 & 0 & 0 \\
D_{3,4} & -0.3641 & -0.9314 & 0.3641 & 0.9314 & 0 & 0 \\
D_{4,5} & 0 & 0 & -0.4362 & -0.8999 & 0.4362 & 0.9427 \\
D_{5,20} & 0 & 0 & 0 & 0 & -0.3337 & -0.9427 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & y_{13} & x_{13} & y_{19} & x_{19} & y_{20} & x_{20} & y_{21} & x_{21} \\
\beta_3 & 0.1452 & 2.362 & -1.0522 & 0.3271 & 0 & 0 & 0 & 0 \\
\beta_5 & 0 & 0 & 0.9070 & -0.5633 & 0 & 0 & 0 & 0 \\
\beta_3 & 0 & 0 & 0 & 0 & 1.0527 & -0.3726 & 0 & 0 \\
\beta_4 & 0 & 0 & 0 & 0 & -0.3641 & -0.9314 & 0 & 0 \\
\beta_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
D_{3,4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
D_{4,5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
D_{5,20} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

The standards we need are: \( \sigma_{CI} = \sigma_{CS} = 0.8 \text{ mm} \) – for laser plumbing, \( \sigma_{STI} = \sigma_{STS} = 0.5 \text{ mm} \) – for tripod on a soft soil \cite{4}, \( \sigma_{\beta,0} = 2.7'' \) – direction-measuring standard with TS7 in one quick look\(^1\). According to \cite{5} the standard for the coordinates of relative positions of given points is \( \sigma_{Y,X} = 50 \text{ mm} \).

Now, on the basis of (15) and (16) one obtains the following measurements standards:
- for angles: 3.0", 3.5", 3.6", 3.5", 3.0",
- for lengths: 2.6 mm, 2.5 mm, 2.5 mm, 2.6 mm.

The application of the least squares method yields:

a) In the classical adjustment, the modified Data Snooping, \cite{2} and \cite{3}, yields:
- for the global test: \( F = 27.046 > 2.506 = F(0.95;3,\infty) \),
- for the local tests: only \( |t_1| = 0.770 < 2.576 = t(0.995) \), and \( |t_2| = 2.517 < 2.576 = t(0.995) \);
thus, all the tests confirm the absolute presence of the gross errors. If it is really the case, we find the answer in the adjustment with taking into account the errors of the given quantities:

b) In the adjustment with taking into account the errors of given quantities, the modified Data Snooping, \cite{2} and \cite{3}, yields:
- for the global test: \( F = 1.897 < 2.506 = F(0.95;3,\infty) \),
- for the local tests: \( \max |t_1| = |t_4| = 2.352 < 2.576 = t(0.995) \).
Thus, all the tests confirm the absence of the gross errors; which has been, of course, to be expected because, for instance, the deviation in longitudinal misclosure is \( f_l = 25 \text{ mm} \), and the transversal misclosure \( f_{\beta} = 42'' \) – which is allowed for all categories of the traverses.

For the coordinate differences, b) minus a), one obtains:
- 16 mm, -21 mm, 4 mm, -9 mm, -3 mm and 2 mm;
thus, the coordinate differences are not negligible.

**Note 1.** For a traverse positioned on three or two given points the method does not yield the perfect results which, certainly, should be studied in more details.

**4 Conclusions**

In the classical adjustment of measurements obtained by using the modern surveying technologies which are related to the given quantities, the Data Snooping test will show that the results contain the gross errors even when they are not really present.

However, when these measurements are adjusted by suggested method which takes into account the influences of the given quantities errors, then the Data Snooping test for the gross errors leads to the null hypothesis that „the results of the measurements contain no gross errors“. For this reason, the mentioned measurements should be adjusted following this methodology.

**References**


